Numerical Aspects in the Simulation of Thermohydraulic Transients in CICC's

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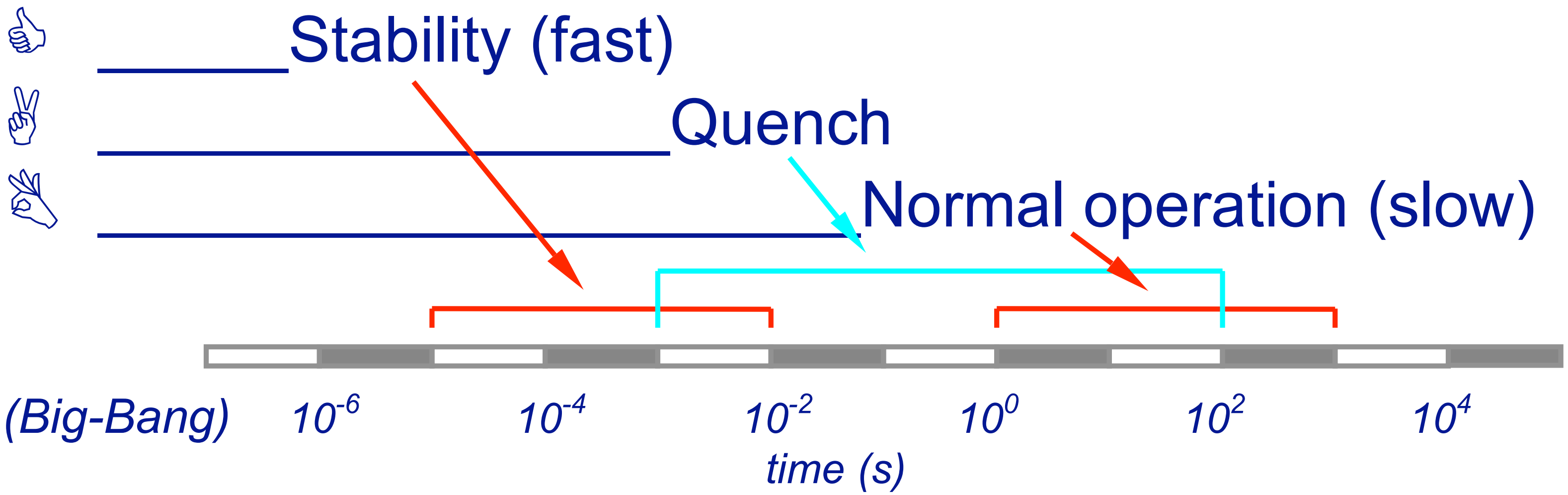
Introduction

What are thermohydraulic transients in CICC's ?



Focus on quench time scale (✌)

☞ Largest time scales span

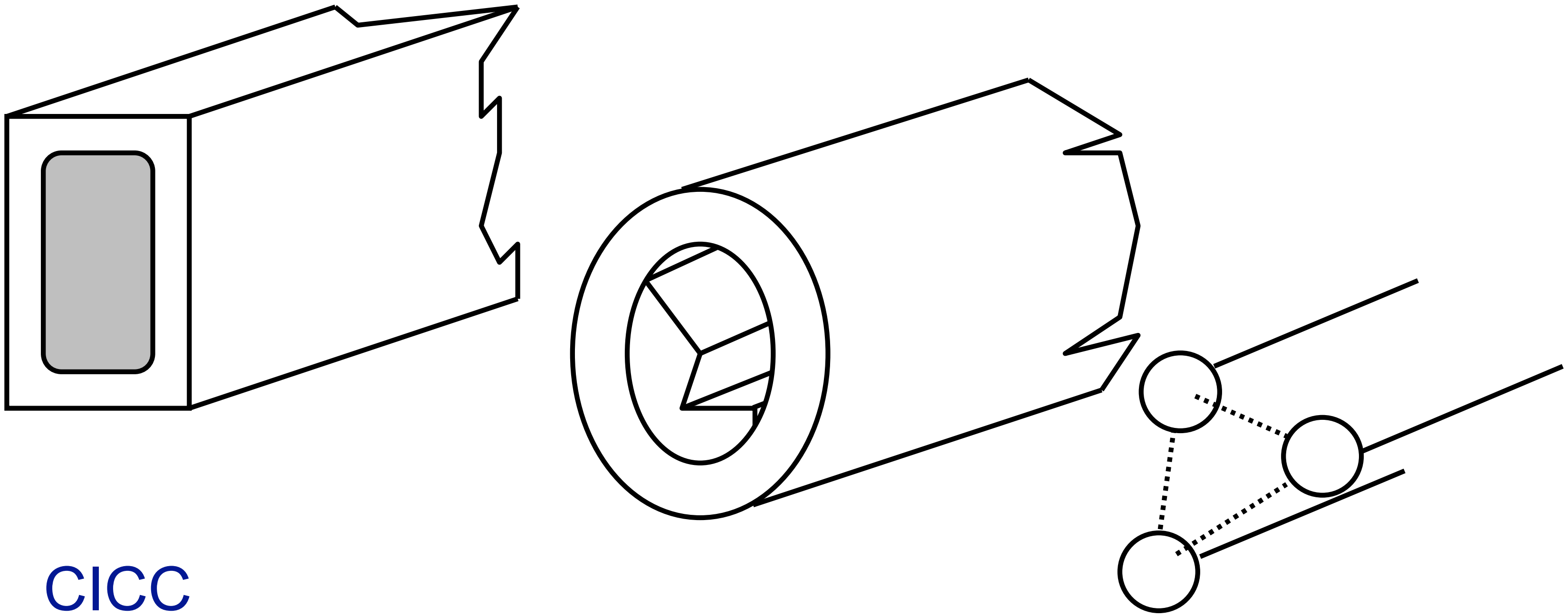
Summary

Quench studies as far back as the 70's ...

Maybe we know everything ?

- Model
- Discuss maths and physics
- Draw consequences for numerics
- Adaptivity
- Conclusions

Model



CICC

Simplified CICC

D.o.F. model

Model

- helium flow

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho v)}{\partial x} = 0$$

$$\frac{\partial (\rho v)}{\partial t} + \frac{\partial (\rho v^2)}{\partial x} + \frac{\partial p}{\partial x} = -2\rho f \frac{v|v|}{D_h}$$

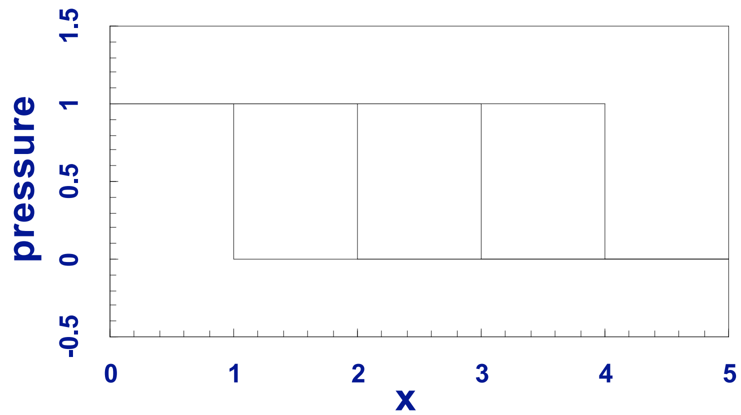
$$\frac{\partial (\rho e)}{\partial t} + \frac{\partial (\rho e v)}{\partial x} + \frac{\partial (p v)}{\partial x} = \sum_i \frac{p_{i,He}}{A_{He}} h_i \left(T_i - T_{He}\right)$$

- conduction

$$\rho_i C_i \frac{\partial T_i}{\partial t} - \frac{\partial}{\partial x}\left(k_i \frac{\partial T_i}{\partial x}\right) = \frac{\dot{q}_i'}{A_i} + \sum_{j, j \neq i} \frac{p_{j,i}}{A_i} h_{j,i} \left(T_j - T_i\right)$$

Physical Characteristics

- Define time scales and characteristic lengths
- Give orders of magnitude
- Sound speed modes (pressure waves, inertia effects)



L ~ 10 (m)

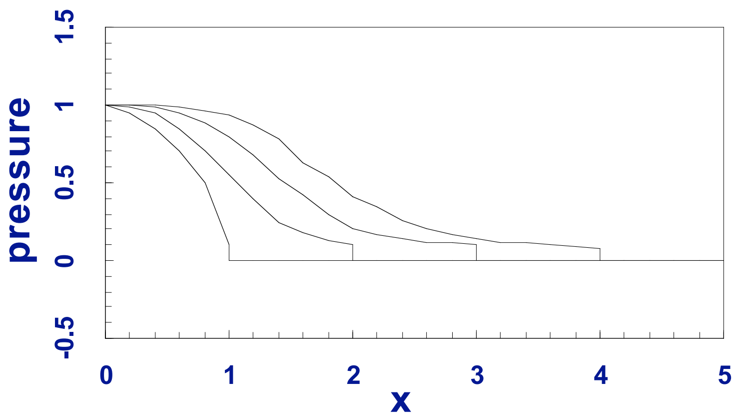
c ~ 250 (m/s)

characteristic time $\tau_s = \frac{L}{c}$ ~40 ms

Physical Characteristics

- Pressure profile (pressure *diffusion*, friction)

$$\frac{\partial p}{\partial x} \approx -2\rho f \frac{v|v|}{D_h} \quad \Rightarrow \quad \frac{\partial p}{\partial t} - \frac{c^2 D_h}{4|v|f}\frac{\partial^2 p}{\partial x^2} + \frac{v}{2}\frac{\partial p}{\partial x} \approx 0$$



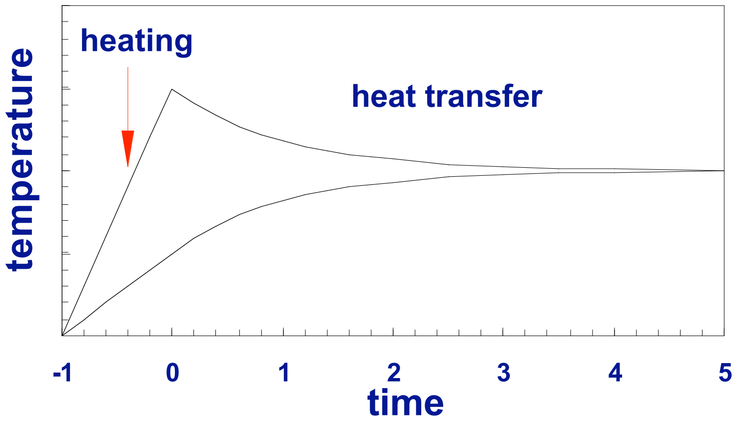
D_h	~	1	(mm)
f	~	0.02	(-)
v	~	5	(m/s)

diffusivity $\alpha_p = \frac{c^2 D_h}{4|v|f}$ ~160 m^2/s

characteristic time $\tau_p = \frac{L^2}{\alpha_p}$ ~600 ms

Physical Characteristics

- Thermal Coupling (heat transfer at wetted surfaces)



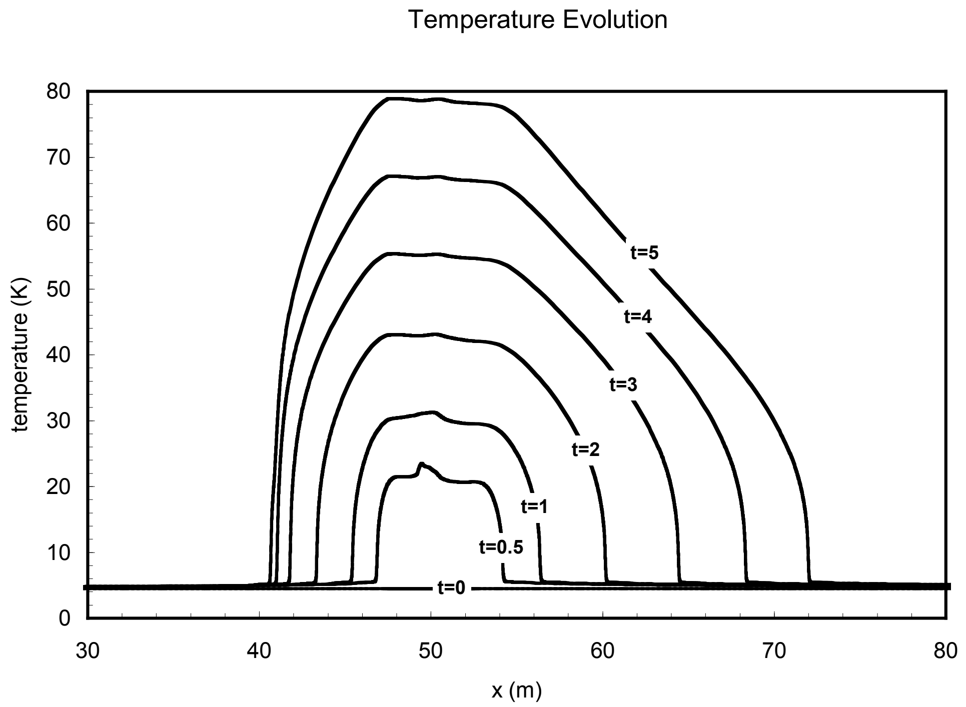
A_{cu}	~	5	(cm^2)
C	~	0.1	(J/Kg K)
ρ	~	8900	(Kg/m^3)
p	~	1	(m^2)
h	~	1000	(W/m^2 K)

characteristic time:

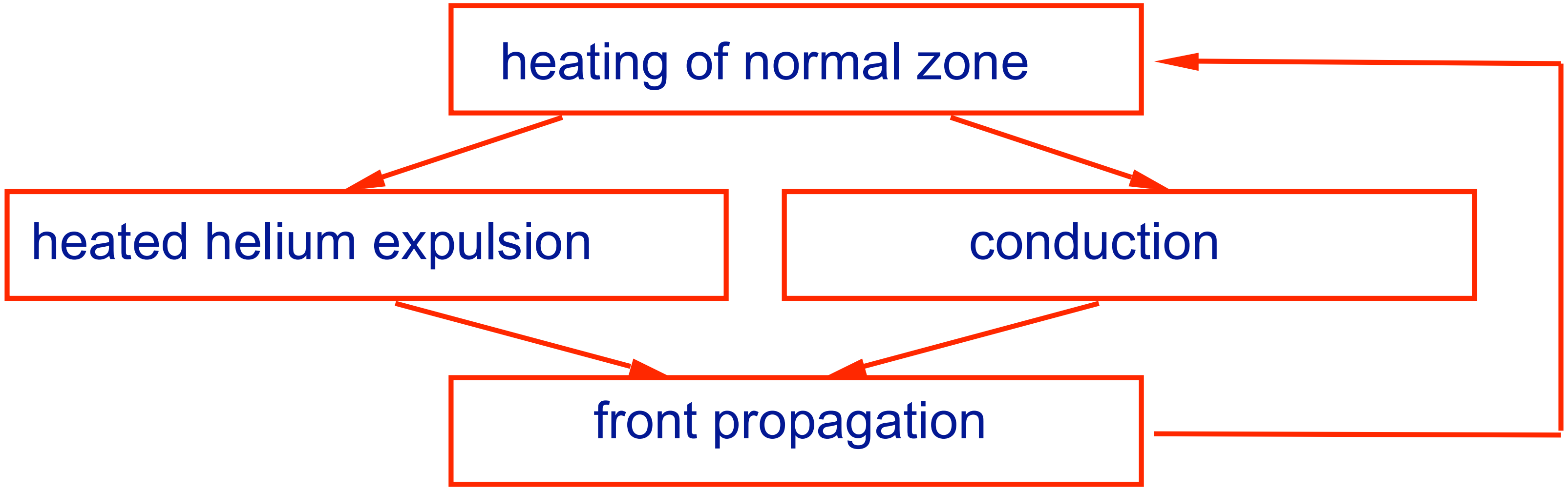
$$\tau_h = \frac{1}{p_{i,j} h_{i,j} \left(\frac{1}{A_i \rho_i C_i} + \frac{1}{A_j \rho_j C_j} \right)} \sim 0.5 \text{ ms}$$

Physical Characteristics

☝👌 Quench Propagation (front movement, free boundary)



Physical Characteristics

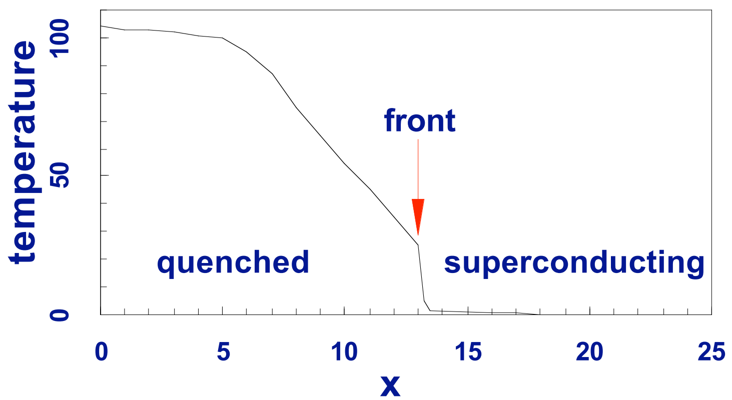


v_q ~ 1-10 (m/s)
L ~ 1000 (m)

characteristic time $\tau_q = \frac{L}{v_q}$ ~ 100-1000 s

Physical Characteristics

- Quench Front Width (boundary layer)



k	~	1000	(W/m K)
v_q	~	1-10	(m/s)
ρ C	~	1.75 10^5	(J/m^3 K)

(weighted values)

characteristic length:

$$\lambda_q \approx \frac{k}{v_q \rho C} \sim 0.5 \text{ mm}$$

Physical Characteristics - Summary

- ☞ Large disparity of time scales
- ☞ Large disparity of characteristic lengths

τ ranging from 0.5 ms to 100 s ⇨ 10^5
λ ranging from 1 cm to 1 km ⇨ 10^5

5 orders of magnitudes are spanned in the time/length scales

Mathematical Characteristics

- Inspect *nature* of the equations
- study problems through mathematically equivalent (but simpler) models
- Hyperbolic system

$$C\frac{\partial u}{\partial t} + A\frac{\partial u}{\partial x} - G\frac{\partial^2 u}{\partial x^2} - Su = q$$

u	unknowns	C	capacity matrix	A	convection matrix
(ρ,v,T), (p,v,T), ...		G	diffusion matrix	S	source matrix
		q	source vector		

Mathematical Characteristics

A model problem

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial x} - \alpha\frac{\partial^2 u}{\partial x^2} = 0$$

incomprehensible defective-confusion equation (Leonard, 1979)

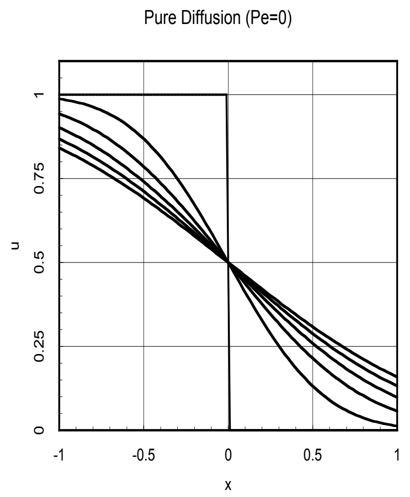
$\alpha \neq 0$ ⇨ parabolic equation

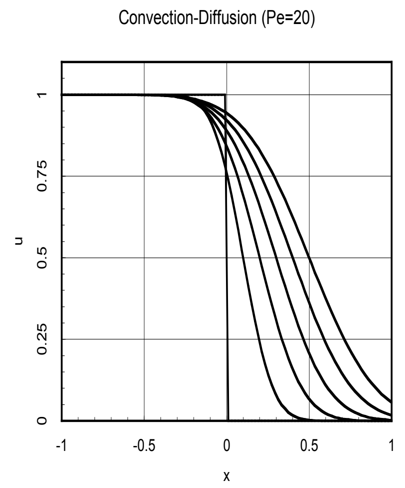
$\alpha = 0$ ⇨ hyperbolic (1st order) equation

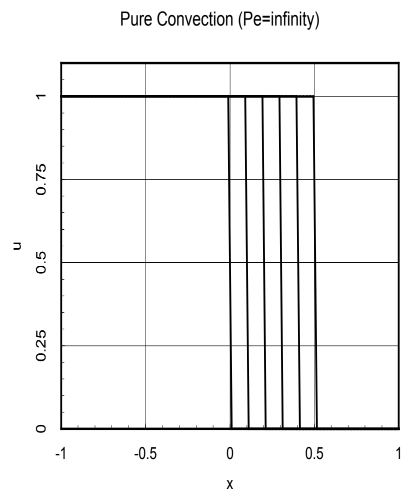
change of *functional class* (from H^1 to H^0) depending on α !

Mathematical Characteristics

Peclet number: $$Pe = \frac{vL}{\alpha}$$







Pe=0	Pe=20	Pe=∞
H^1	H^1	H^0

Mathematical Characteristics - Spaces

H^0 space of those functions that are *square-integrable*

H^1 space of those functions that are *square-integrable* and whose first derivative is also *square-integrable*

H^0 is a box of apples and pears

H^1 is a box of apples only

H^0 is <u>wider</u> than H^1

Mathematical Characteristics

Which equations show hyperbolicity ?

- helium flow

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v)}{\partial x} = 0$$

$$\frac{\partial(\rho v)}{\partial t} + \frac{\partial(\rho v^2)}{\partial x} + \frac{\partial p}{\partial x} = -2\rho f \frac{v|v|}{D_h}$$

dominated by friction

$$\frac{\partial(\rho e)}{\partial t} + \frac{\partial(\rho e v)}{\partial x} + \frac{\partial(p v)}{\partial x} = \sum_i \frac{p_{i,He}}{A_{He}} h_i \left(T_i - T_{He}\right)$$

- conduction

$$\rho_i C_i \frac{\partial T_i}{\partial t} - \frac{\partial}{\partial x}\left(k_i \frac{\partial T_i}{\partial x}\right) = \frac{\dot{q}_i'}{A_i} + \sum_{j,j \neq i} \frac{p_{j,i}}{A_i} h_{j,i}\left(T_j - T_i\right)$$

weak coupling

Mathematical Characteristics

✌ Moving Boundary

Implicit moving boundary equation $T=T_{cs}$

A model problem for pure convection

$$\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial x} = \gamma H\left(T - T_{cs}\right)$$

$$\frac{\partial}{\partial t}\left[\int_0^{X_q(t)} \rho \, dx\right] = \frac{\partial X_q(t)}{\partial t}\rho\left(X_q(t)\right) + \int_0^{X_q(t)} \frac{\partial \rho}{\partial t} dx = 0$$

Mathematical Characteristics

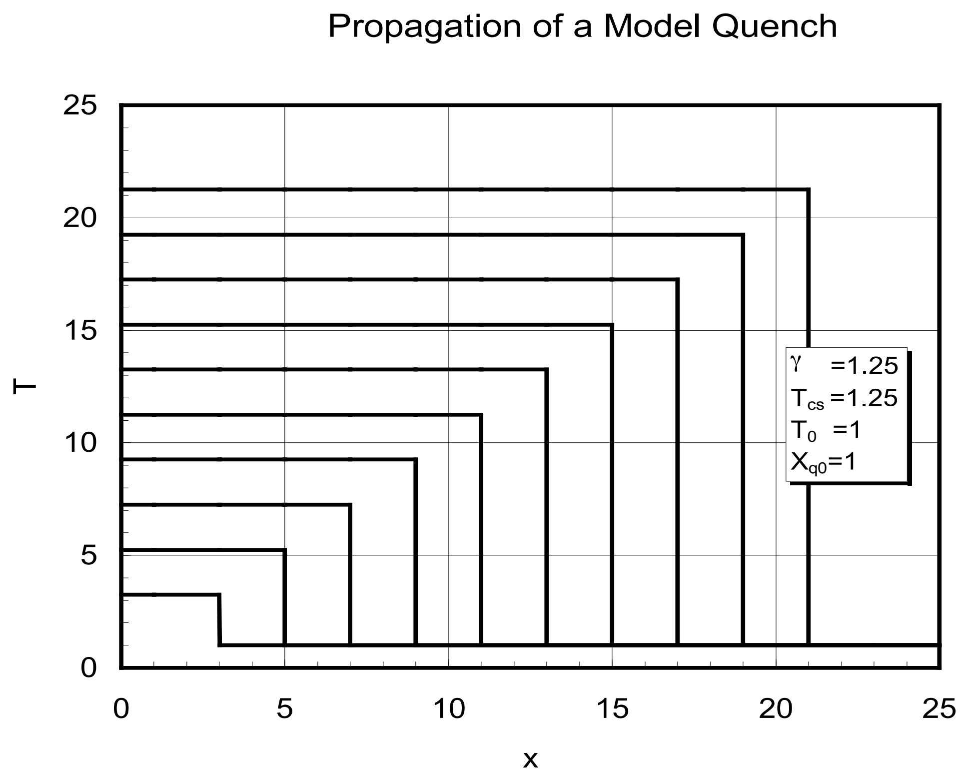
Analytic solution:

moving boundary with speed $v_q = \frac{X_{q0}\gamma}{T_{cs}}$

increasing temperature with constant rate $\frac{\partial T}{\partial t} = \gamma$

- ☞ v_q is a constant
- ☞ dependence on X_{q0} and γ (integral of source)
- ☞ the system is meta-stable (thermal runaway)

Mathematical Characteristics



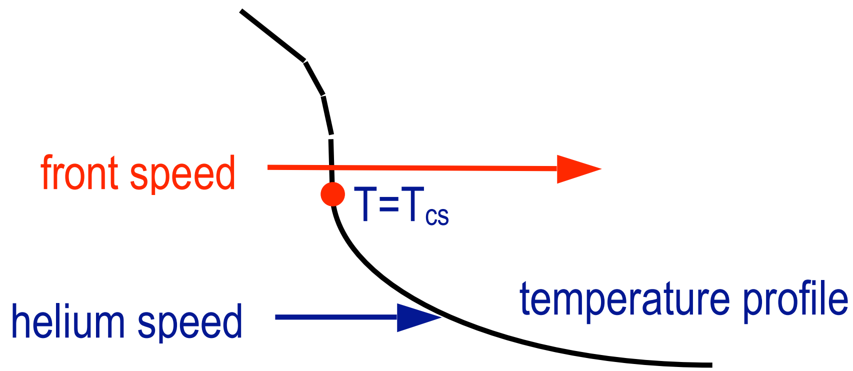
Mathematical Characteristics

Effect of diffusion (perturbation to model problem):

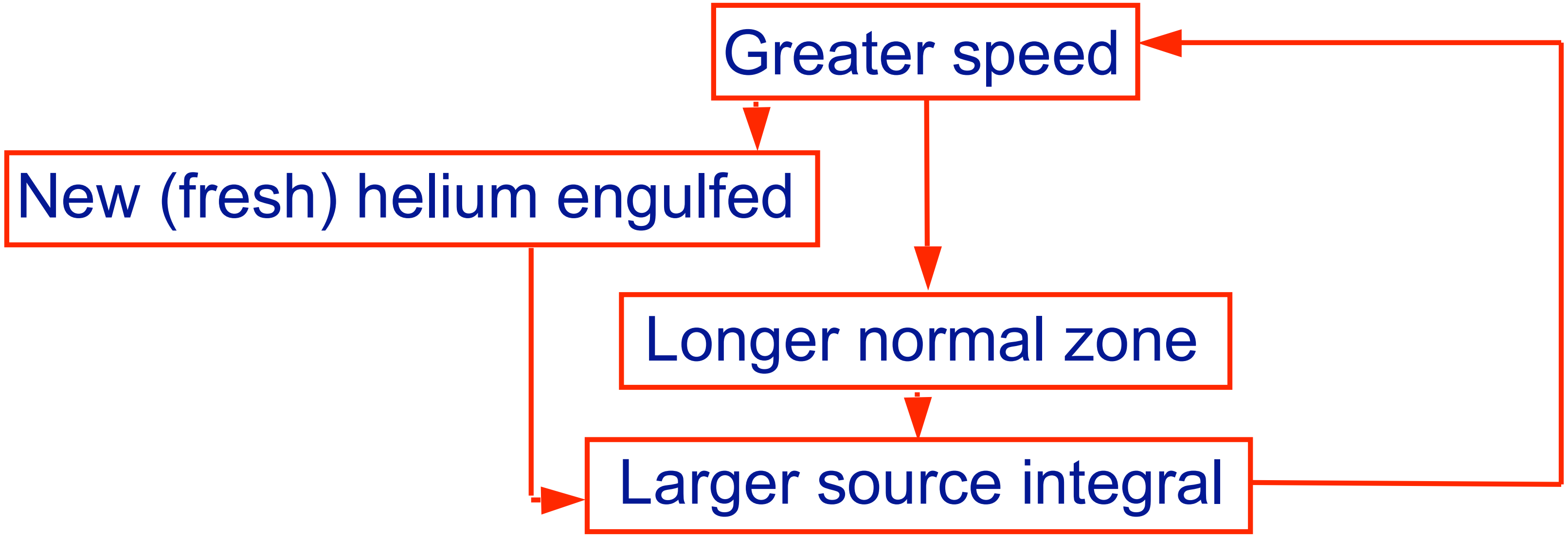
$$\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial x} - \alpha\frac{\partial^2 T}{\partial x^2} = \gamma H\left(T - T_{cs}\right)$$

Additional front speed

$$v_{ad} = \sqrt{\frac{\alpha\gamma}{\left(T_{cs} - T_o\right)}}$$



Mathematical Characteristics



☞ Quench propagation at increasing speed

Mathematical Characteristics - Summary

- strong hyperbolic character for energy balance
- sharp temperature fronts associated to both:
  - hyperbolic character
  - free boundary
- *meta-stable* system sensitive to perturbations
- pressure fronts are not expected (no shocks)
- problem is *stiff* (large time scale disparity)

Numerics

Facts which have the *strongest* consequences:

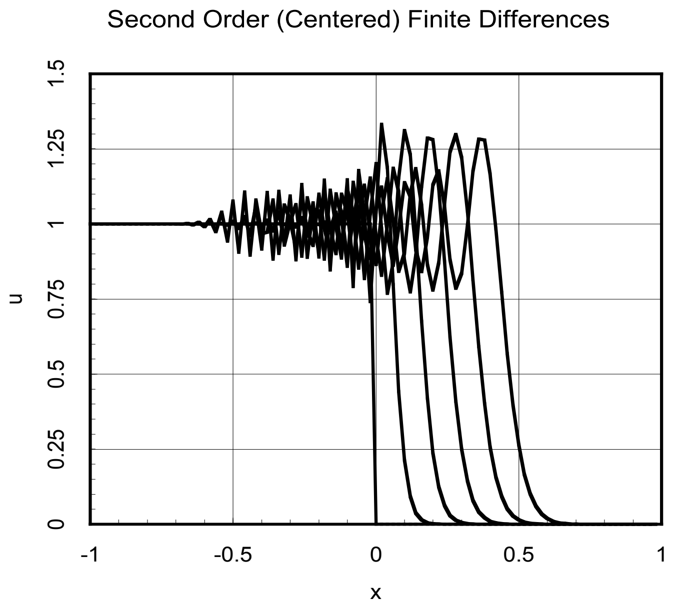
- Hyperbolicity (significant 1st order space derivative term)
- Non-linearity, meta-stability and moving boundary
- Stiffness

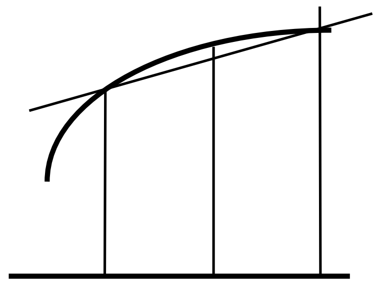
Concentrate on FD's and FE's (most flexible methods)

Remarks on other methods (few)

Numerics

- Hyperbolicity
  First trial (choice for parabolic ode's) central differences

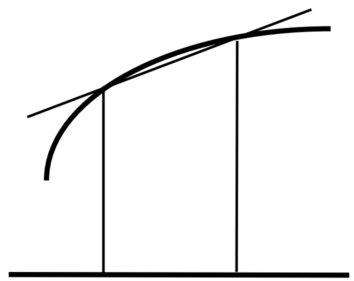




$$\frac{\partial u}{\partial x} \approx \frac{u_{n+1} - u_{n-1}}{2\left(x_{n+1} - x_{n-1}\right)}$$

Numerics

A step back (*physics of flow ?*) upwind differences



$$\frac{\partial u}{\partial x} \approx \frac{u_{n+1} - u_n}{x_{n+1} - x_n}$$

Numerics

Central differences in x & t ⇨ 2nd order of accuracy

$$\varepsilon = o\left(\Delta x^2, \Delta t^2\right)\frac{\partial^3 u}{\partial x^3}$$

dispersion

Upwind differences in x & t ⇨ 1st order of accuracy

$$\varepsilon = o\left(\Delta x, \Delta t\right)\frac{\partial^2 u}{\partial x^2}$$

diffusion

Phenomenological...Can we find a *better* explanation ?

Numerics

- FE (Galerkin) ≡ to FD central differences in x & t
- FE (Galerkin) looks for a solution in H^1 (in the apple box)
- the solution is in H^0 (apple+pears box, not an apple, a pear)

Find the apple that, in a weak sense, gives the best approximation to the pear

A bigger box (more nodes)? ⇨ A better approximation

<u>An apple will never be a pear</u>

Numerics

- ☞ FE (Petrov-Galerkin) ≡ to FD upwind differences in x & t
- ☞ FE (Petrov-Galerkin) is optimal (1-D, steady state)
- ☞ No optimal treatment for general transient case

Numerics

- Moving boundary (perturbed model problem)

$$\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial x} - \alpha\frac{\partial^2 T}{\partial x^2} = \gamma H(T - T_{cs})$$

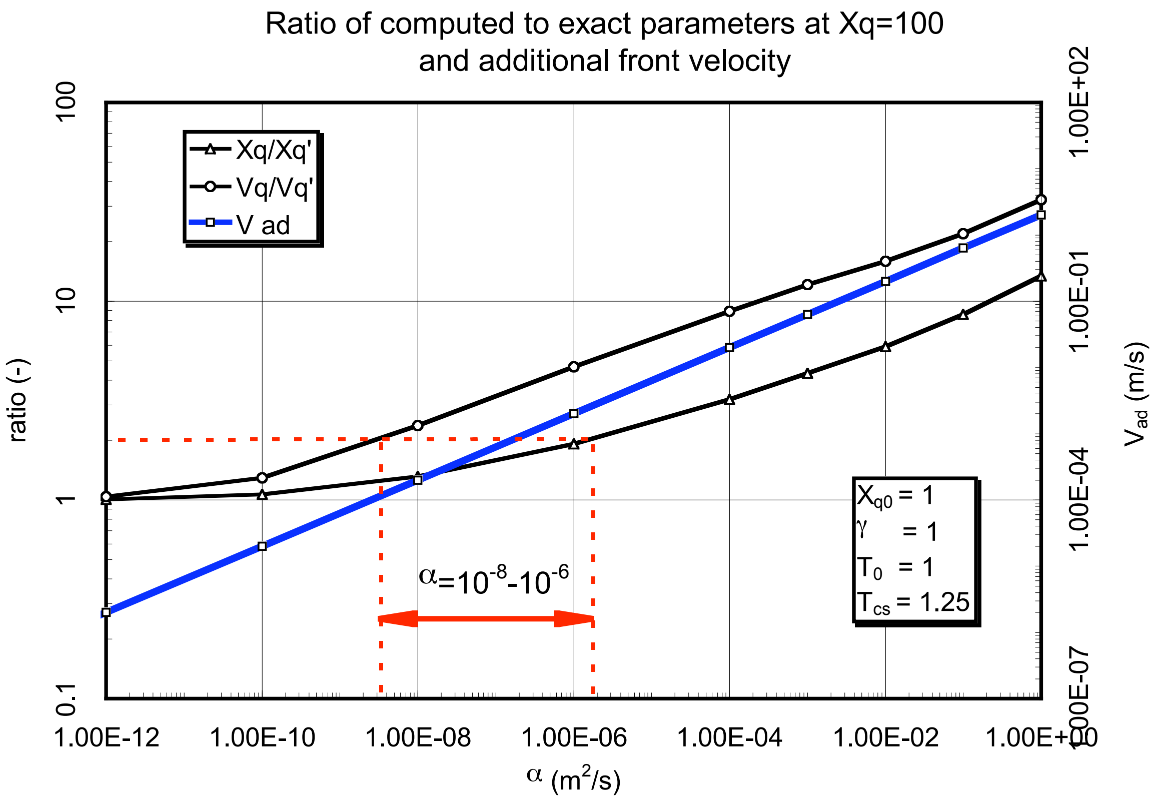
1st order method:

$$\frac{\partial u}{\partial t} \approx \frac{u^{n+1} - u^n}{t^{n+1} - t^n} \qquad \frac{\partial u}{\partial x} \approx \frac{u_{i+1}^{n+1} - u_i^{n+1}}{x_{i+1} - x_i}$$

- *numerical diffusion*

$$\alpha = v\left(\frac{\Delta x}{2} + \frac{v\Delta t}{2}\right)$$

Numerics



Ratio of computed to exact parameters at Xq=100
and additional front velocity
Xq/Xq'
Vq/Vq'
V ad
ratio (-)
100
10
1
0.1
1.00E-12
1.00E-10
1.00E-08
1.00E-06
1.00E-04
1.00E-02
1.00E+00
α (m²/s)
1.00E+02
1.00E-01
1.00E-04
1.00E-07
V ad (m/s)
α=10^-8-10^-6
Xq0 = 1
γ = 1
T0 = 1
Tcs = 1.25

Numerics

desired $\alpha = 10^{-8}$-10^{-6}

assume $\Delta x \approx v\Delta t$ (step at C=1) and $v \approx 1$ m/s

⇩

$\Delta x \approx \Delta t \approx 10^{-8} - 10^{-6}$!

Length: 100 m

⇩

Mesh: 100 M_{Nodes}

Time span: 10 s

⇩

Time steps: 10 M_{Steps}

$CPUtime_{(1\ CPU_{\mu}s/Node\ Step)}$: 10^{9} CPUs ⇨ 32 CPUy

☺

Numerics - Adaptivity

What is adaptivity ?

- Define an error estimator (definition of ε)
- Build a *mesh designer* (Δx, Δt based on ε)
- Re-mesh until $\varepsilon < \varepsilon_{max}$

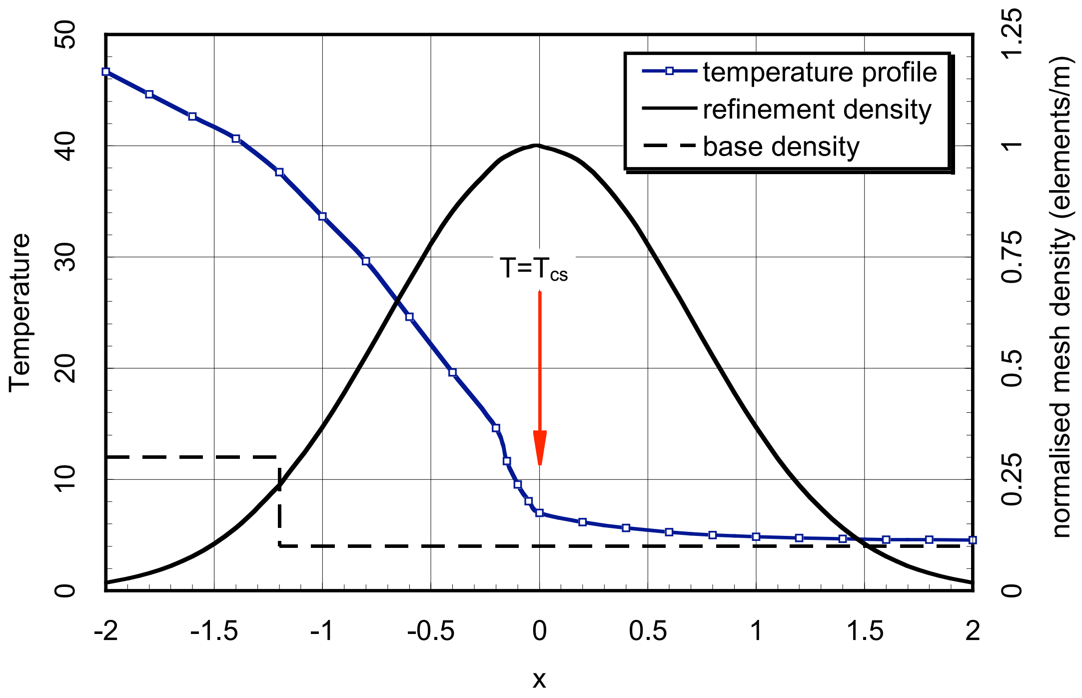
What are the problems with respect to quench simulation ?

- No *a priori* definition of ε
- Re-meshing (may) imply iterations (non-linearity in transient)

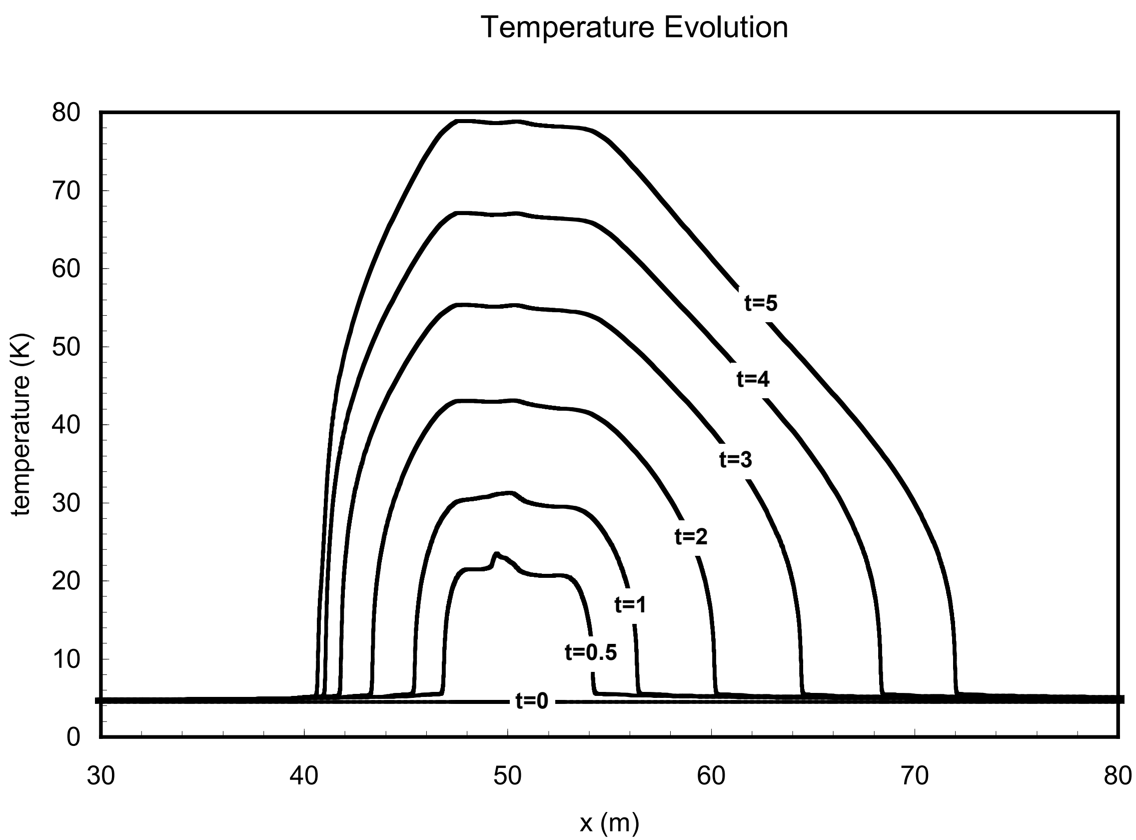
An alternative: front tracking

Numerics - Adaptivity

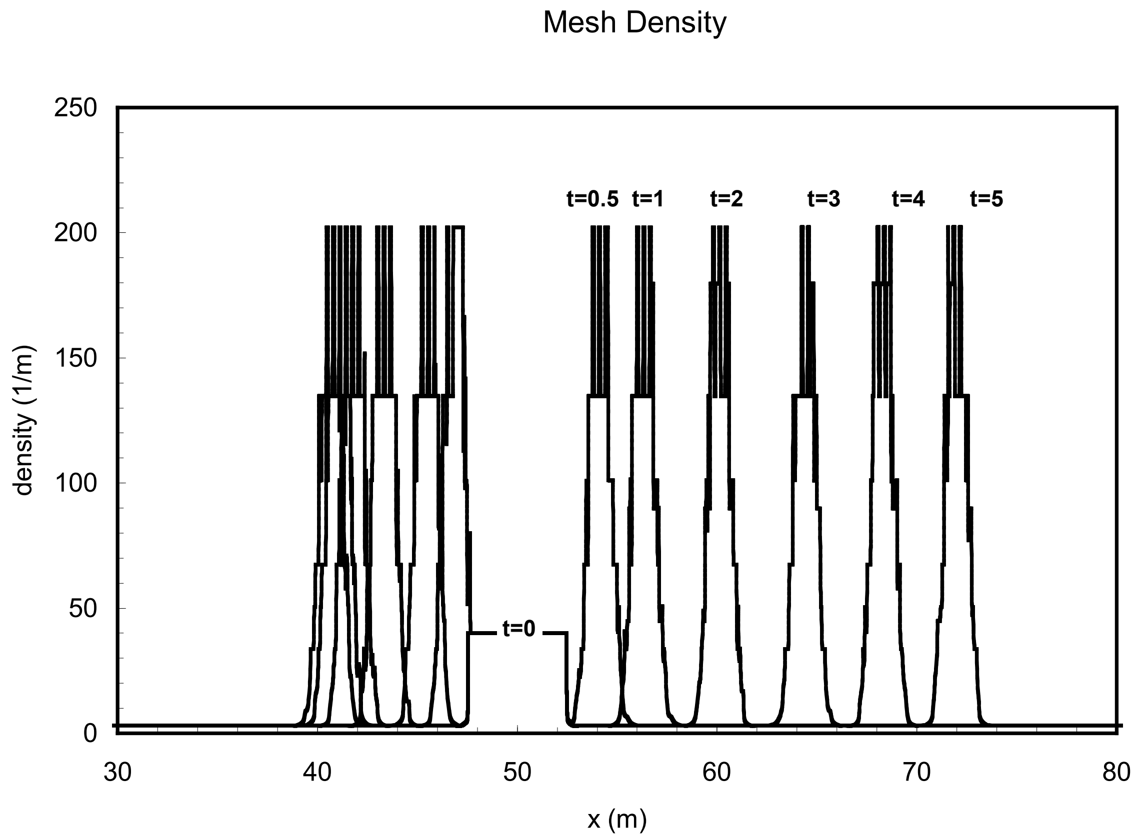
Front tracking to determine desired mesh density



Numerics - Adaptivity



Numerics - Adaptivity



Numerics

👌 Stiffness

An example:

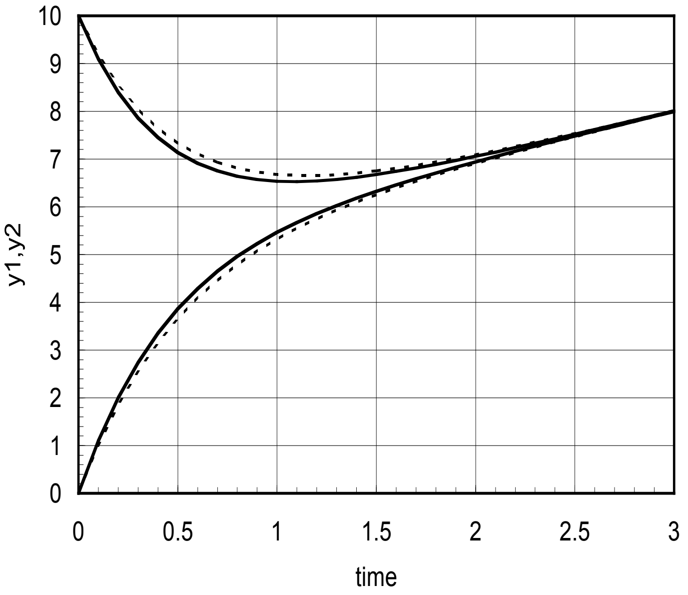
$$\begin{cases} \dfrac{\partial y_1}{\partial t} + h\left(y_1 - y_2\right) = S \\ \dfrac{\partial y_1}{\partial t} + h\left(y_2 - y_1\right) = S \end{cases}$$

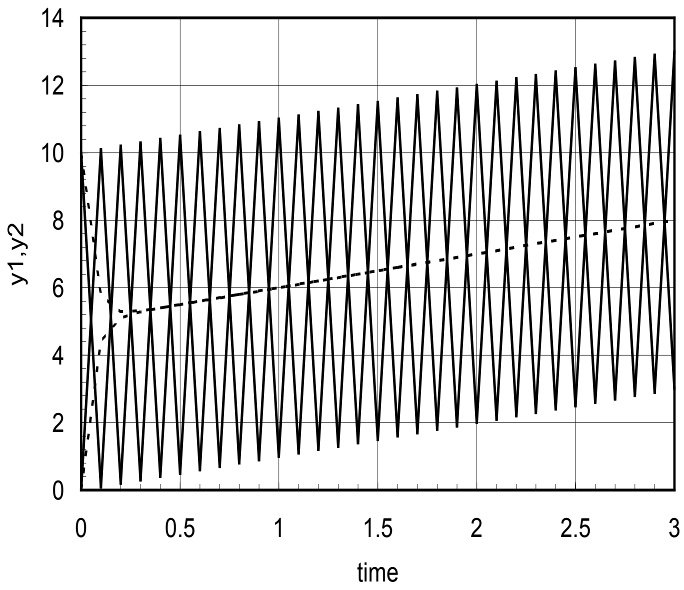
☞ 2 times scales

Δy ⇨ $\tau=1/2h$ <y> ⇨ $\tau=S/\langle y \rangle$

Numerics

Explicit integration (Euler forward)





h=1 *S=1* *Δt=0.1*

h=10 *S=1* *Δt=0.1*

Numerics

What are the *fast* modes (compared to τ_q) ?

- ☞ pressure waves
  (damped by friction)

- ☞ temperature differences in the cable cross section
  ($T_{He} \approx T_{Co}$)

Implicit treatment required

- ☞ non-conservative form of flow equations (p,v,T)

Numerics

☝👌 Other methods, packages, miscellanea

- Eulerian vs Lagrangian method (moving FE's)
- Packages
  solvers for systems of PDE's ⇨ scarce
  solvers for systems of ODE's ⇨ many for 2nd order
  ? generality ?
- Other methods ?

Numerics - Summary

- *Order* of method (critical)
  - first order for stability (and damping)
  - second order for improved accuracy (a must !)
- Adaptivity
  - improves accuracy at given computational work
  - p-h adaptivity *optimal*
- Implicit solution
  - suppression of fast (useless) modes
  - retained for generality (waves, temperature gradients)

Conclusions

Didn't we think we knew everything ?

☞ physical implications (compressible flow in pipes)

☞ mathematical implications (moving-boundary)

☞ numerical implications (methods' development)