A Practical Fit for the Critical Surface of NbTi

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Abstract—Known expressions for the critical temperature, critical field and Pinning force in NbTi are combined into a self-consistent fit formula that provides the critical current density as a function of temperature and field. The main advantage of such a fit is the extended validity range. Data available in literature and measurements on LHC strands are used to demonstrate the accuracy of the fit. The J_c data-sets used to cover a range of field from 0 T to 9 T and temperature from 1.9 K to 9 K. The standard deviation of the fits presented is of the order of 5 % or better. This accuracy is generally sufficient for design purposes, extrapolation and scaling of measured results. Better accuracy, e.g. for short sample limit prediction, can be achieved restricting the domain of validity.

I. INTRODUCTION

Fitting formulae for the critical surface J_c (B,T) of NbTi are commonly used for the engineering calculation of short sample limit, temperature and current margins, and for the calculation of magnetization. Several practical fits have been produced in the literature, see for example [1]-[4]. The main drawback of these fits is that often they have been tailored to restricted ranges of field B and temperature T, to a particular strand material, or are defined as complex piecewise polynomial fits. Surprisingly, the consistent use of a Pinning model and a temperature scaling law seems to have eluded common practice in magnet design. The purpose of this paper is to resort back to known expressions for the critical temperature, critical field and Pinning force density to produce a simple but general fit of the critical surface of commercial NbTi. I will show by comparison with published data that in spite of its simplicity this approach has a good engineering value.

II. CRITICAL SURFACE FIT

Table I reports the symbols used in the paper for the critical parameters. It is useful to introduce the reduced temperature t and the reduced field b defined as follows:

$$t = \frac{T}{T_{c0}} \tag{1}$$

$$b = \frac{B}{B_{c2}(T)} \cdot \tag{2}$$

In accordance with the results of Ekin on NbTi alloys [5], we neglect the dependence of all parameters on intrinsic

strain. The function chosen here for the fit of the critical surface as a function of the reduced parameters t and b is given by:

$$J_c = \frac{C_0}{B} b^\alpha (1-b)^\beta (1-t^n)^\gamma \cdot \tag{3}$$

Equation (3) has 4 free parameters: a normalization constant C_0, two parameters describing the dependence on the reduced field, α and β, and a parameter describing the dependence on the reduced temperature, γ. Note that J_c in (3) depends on temperature explicitly through the last term and implicitly through the critical field dependence which enters the definition of the reduced field. A suitable critical field dependence on temperature can been taken from Lubell [1]:

$$B_{c2} = B_{c20} (1-t^n) \tag{4}$$

where the exponent $n = 1.7$ appears to provide a satisfactory fit to most alloy compositions. For completeness, we recall that the critical temperature dependence on field is obtained as the inverse of the critical field relation (4):

$$T_c = T_{c0} \left(1 - \frac{B}{B_{c20}} \right)^{\frac{1}{n}} \cdot \tag{5}$$

The final parameter of interest for practical use is the current sharing temperature T_{cs}. This is defined implicitly as the temperature at which the operating current density in NbTi J_{op} equals the critical current density, or:

$$J_{op} = \frac{C_0}{B} b^\alpha (1-b)^\beta (1-t_{cs}^n)^\gamma \tag{6}$$

where t_{cs} is the reduced current sharing temperature:

$$t_{cs} = \frac{T_{cs}}{T_{c0}} \cdot \tag{7}$$

TABLE I
CRITICAL FUNCTIONS AND PARAMETERS

$J_c(B,T)$	[A/m^2]	critical current density
$T_c(B)$	[K]	critical temperature
$B_{c2}(T)$	[T]	upper critical field
T_{c0}	[K]	maximum critical temperature (at B=0)
B_{c20}	[T]	maximum upper critical field (at T=0)
$F_p(B,T)$	[N/m^3]	Pinning force density
$T_{cs}(B,J_{op})$	[K]	current sharing temperature

Equations (6) and (7) can be solved numerically (e.g. by an iterative bisection method), providing an arbitrarily accurate value of the current sharing temperature.

Equation (3) has a well-established structure that can be derived as a direct consequence of the temperature scaling law of Fietz and Webb [6]-[7]. Using the definition of the critical field given by (4) we can rearrange the terms in (3) and express the Pinning force density $F_p = J_c \times B$ in the following manner:

$$F_p = J_c \times B = f(b)B_{c2}^\gamma \qquad (8)$$

where $f(b)$ is the Pinning curve of the material [7] that depends on the reduced field only, and the temperature scaling through B_{c2} is indeed in accordance with [6]. By comparison of (3) and (8) we see that the Pinning curve for the material $f(b)$ assumed in the fit is given by:

$$f(b) = \frac{C_0}{B_{c20}^\gamma} b^\alpha (1-b)^\beta \cdot \qquad (9)$$

It is generally more useful to define a normalized Pinning curve:

$$\frac{f(b)}{f_{max}} \propto b^\alpha (1-b)^\beta \qquad (10)$$

that exhibits a maximum at $b_{max} = \alpha/(\alpha+\beta)$ [8]. The above functional dependence comes from standard practice in the analysis of Pinning data from J_c measurements [5]-[7]-[8]. Equation (10) shows that the two exponents α and β are solely related to the dependence of the normalized Pinning force on the reduced field, while the exponent γ is determined by the temperature scaling law as expressed by (8).

III. APPLICATIONS TO PUBLISHED Jc DATA SETS

The validity of the fit can only be confirmed by direct comparison with measurements. To demonstrate the accuracy achievable with the formulae given above we have fitted data sets of NbTi J_c data. Most of these data are the same as collected by Green in Ref. [3]. In particular we have chosen in Ref. [3] the data from Spencer [9], Somerkoski [10] and Taylor [11]. In addition the data set used by Morgan [4] and Hudson [12] have been tested. They refer to different strands, produced during a time span of about 10 years, with different alloy composition and very wide variation of values for J_c. In all cases the value of the critical current density J_c has been normalized to the measurement at 5 T and 4.2 K for convenience of representation.

The data set of Spencer [9] refers to a Nb-46.5Ti alloy tested at fields in the range 0 T to 8 T and temperatures in the range 4.2 K to 9 K. A total of 33 measured points have been fitted, with the results shown in Fig. 1. The maximum local error, relative to the reference value of J_c at 4.2 K, 5 T, is
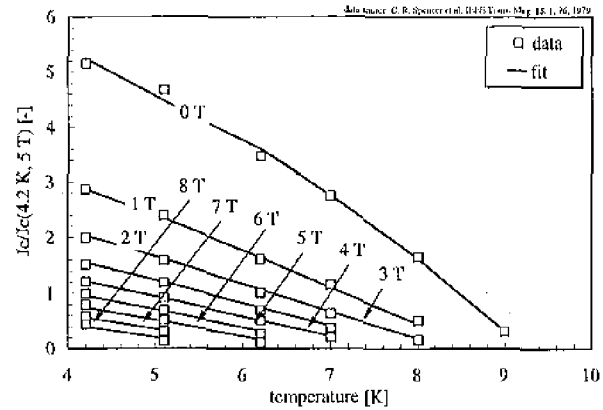

Fig. 1. Temperature dependence of the normalized $J_c(B,T) / J_c$ (5 T, 4.2 K) as reported by Spencer [9] on Nb-46.5Ti wire, and fitted using (3). Field values are indicated on the plot.
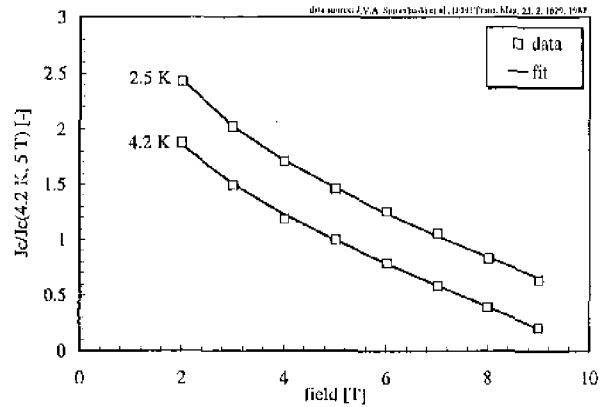

Fig. 2. Field dependence of the normalized $J_c(B,T) / J_c$ (5 T, 4.2 K) as reported by Somerkoski [10] on Nb-46.5Ti wire, and fitted using (3). Temperature values are indicated on the plot.

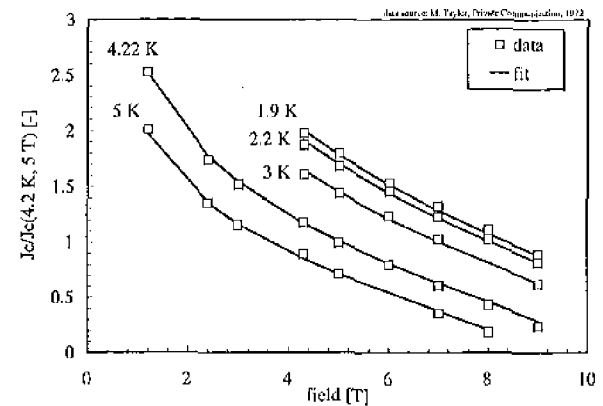Fig. 3. Field dependence of the normalized $J_c(B,T) / J_c$ (5 T, 4.2 K) as reported by Taylor [11] on Nb-44Ti wire, and fitted using (3). Temperature values are indicated on the plot.

approximately 20 % and is found at zero background field. The overall standard deviation of the fit is about 5 % (again referred to the reference value of J_c (4.2 K, 5 T)).

The second data set, collecting data from Somerkoski [10] also refers to a Nb-46.5Ti alloy tested at fields in the range of 2 to 9 T and temperatures of 2.5 and 4.2 K. The data set consists of 16 points. The result of the fit and the fit parameters used are reported in Fig. 2. The maximum local error achieved is of the order of 3.5 % while the standard deviation is about 1.5 %.

The third data set is from Taylor [11], for a Nb-44Ti alloy tested at fields in the range of 1 to 9 T and temperatures of 1.9, 2.2, 3, 4.22 and 5 K. The data set consists of 35 points. The results are reported in Fig. 3. The maximum local error achieved is of the order of 3.5 % and the standard deviation is about 1.9 %.

The data of Morgan [4] for SSC NbTi wire consists of 44 points in the field range of 4 to 8 T and temperature range of 2.5 to 7.5 K. Results on the fit are presented in Fig. 4.
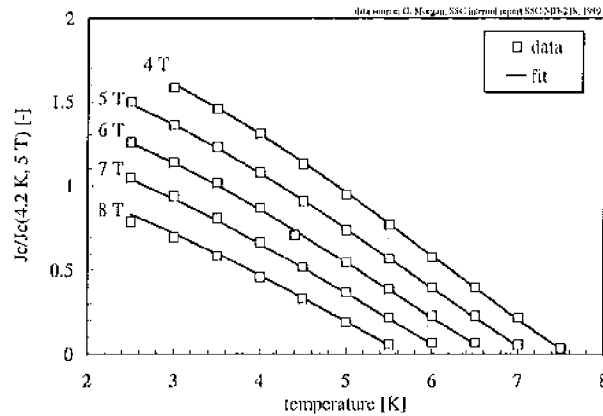

Fig. 4. Temperature dependence of the normalized J_c (B,T) / J_c (5 T, 4.2 K) as reported by Morgan [4] on SSC NbTi wire, and fitted using (3). Field values are indicated on the plot.
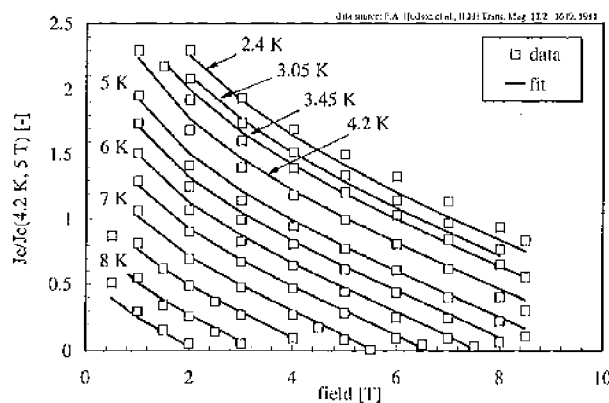

Fig. 5. Field dependence of the normalized J_c (B,T) / J_c (5 T, 4.2 K) as reported by Hudson [12] on Nb-46.5Ti IMI wires, and fitted using (3). Temperature values as indicated on the plot. In the interval 5 to 8.5 K the spacing of the curves is 0.5 K.

source		Ref.[9]	Ref.[10]	Ref.[11]	Ref.[4]	Ref.[12]
Ti content	[%]	46.5	46.5	44	na	46.5
points	[-]	33	16	35	44	88
B_{c20}	[T]	14.5	14.2	14.4	14.4	14.25
T_{c0}	[K]	9.2	8.5	8.9	9.2	9.35
C_0	[T]	23.8	28.6	28.5	37.7	28.4
α	[-]	0.57	0.76	0.64	0.89	0.80
β	[-]	0.90	0.85	0.75	1.10	0.89
γ	[-]	1.90	1.76	2.30	2.09	1.87
ε_{max}	[%]	20	3.5	4.1	1.5	17.1
σ	[%]	5	1.5	2.3	1.2	5.0

Maximum local error is below 1.5 %, and standard deviation is 1.2 %.

Finally, the data of Hudson [12] refers to Nb-46.5Ti wire produced by IMI, and consists of 88 points in the field range of 0.5 to 9 T and temperature range of 2.5 to 8.5 K. This is the data set with the largest field and temperature extension tested. Results of the fit are presented in Fig. 5. Maximum local error is 17 % (again at very low field, below 1 T, and high temperature, above 8 K). At low temperature (2.4 K) and high field (above 6 to 8 T) the local error ranges from 8 to 12 %. The overall standard deviation of the fit is 5 %.

Table II contains a summary of the number of points used for each data set, the fit parameters, the maximum local error ε_{max} and the standard deviation σ. Note that the value quoted for C_0 refers to the fits of normalized J_c, and has therefore units of [T], instead of [A T/m^2] as can be derived from (3). The typical standard deviation for the complete data set is always in the range of 2 to 5 %, with maximum errors in the range of 5 %. Higher errors, of the order of 20 %, should be expected at the boundary of the fitting range, especially at low field. Comparing the values of the fitting parameters α, β and γ we see that their range of variation is restricted. In particular α is of the order of 0.5 to 0.8, implying an approximate $B^{-1/2}$ dependence for J_c at low field, as indeed found from J_c and magnetization measurements [2]. The second parameter, β, has values close to 1. Finally, the third parameter, γ, is in the range of 2.

The values of B_{c20} and T_{c0} were not optimized in the fits. Measured or quoted data for the material were used if available in the references. For the cases where no values were quoted the choice was restricted in order to remain consistent with the typical material characteristics reported in the literature (B_{c20} around 14 to 14.5 T and T_{c0} around 9 K). Better tuning of the fits would be possible considering also these parameters as free, and adjusting them generally to lower values than those taken above. In this case however the physical meaning of the upper critical field and temperature would be lost, and therefore this way was not pursued further.

Using (9) it is possible to compute the normalized Pinning curve for each data set fitted. The curves obtained are reported in Fig. 6, and are compared to data reported by Larbalestier [7] representing the typical behaviour of a Nb-46.5Ti alloy. Note that this data can be fitted using (9) with a choice of $\alpha = 0.65$ and $\beta = 1$. As shown in Fig. 6 the
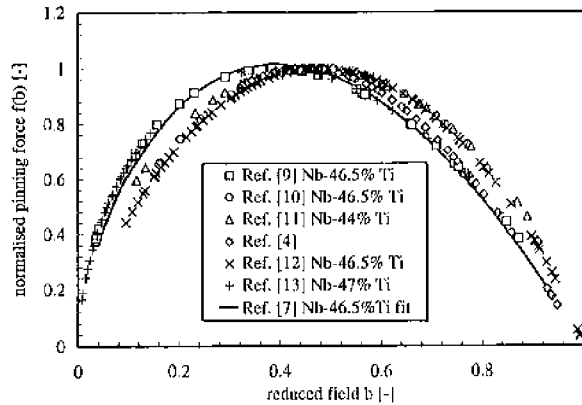
Fig. 6. Normalized Pinning force as computed from the critical current fits reported here. The values are compared to a data fit quoted in Ref. [7].
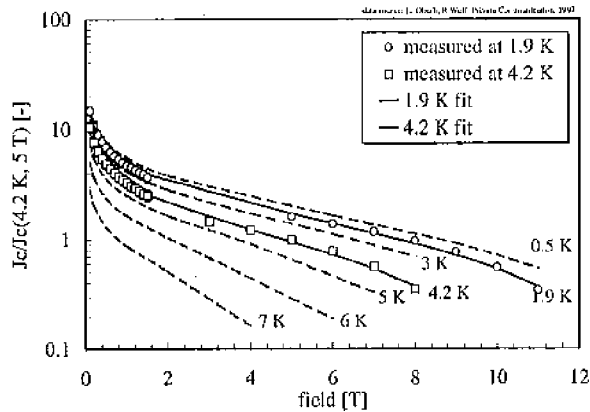

Fig. 7. Field dependence of the normalized $J_c(B,T) / J_c$ (5 T, 4.2 K) for a typical LHC strand [13]-[14], measured at 1.9 and 4.2 K. The fit to the data is shown (solid lines) together with curves generated for different temperatures (dashed lines).

Pinning curves obtained are in reasonable agreement with the expected behaviour. We recall that the shape of the Pinning curve is alloy and process specific, and is not a material characteristic. Therefore variations of the Pinning curve must be expected among different strands.

IV. EXAMPLE OF JC SCALING FOR AN LHC STRAND

An example of practical application of the fitting formula is shown in Fig. 7. The J_c data at 1.9 K and 4.2 K is relative to a typical Nb-47Ti strand for the cable of the outer layer of an LHC main bending dipole. Data at high field (above 3 T) are derived from I_c measurements [13], while the data at low field (below 1.5 T) are deduced from magnetization measurements on the same strand [14]. The error on the data at low field can be large (of the order of 10 %) because of the uncertainty on the details of filament magnetization.

The fit was obtained taking $B_{c20} = 14.5$ T and $T_{c0} = 9.2$ K. The fit parameters are $C_0 = 27.04$ T, $\alpha = 0.57$, $\beta = 0.9$ and

$\gamma = 2.32$. The standard deviation of the fit is approximately 5 %. The maximum local error (at low field) is 11 %, while at high field the local error is of the order of 1 to 5 %. The Pinning curve for the strand material is reported in Fig. 6 and shows an excellent agreement with the expected behaviour of the alloy. In addition to the fitted data Fig. 7 shows the J_c curves generated for different temperatures.

V. CONCLUSIONS

The use of a simple but consistent Pinning model and temperature scaling for the critical surface of NbTi appears to provide a satisfactory match to several J_c data sets on different strands. The resulting fit is valid over the full range of field and temperature. Typical overall fitting accuracy is of the order of 5 % and better, with maximum local errors of the order of 20 % (mostly in the low field region, below 1 T). These values are adequate for comparisons among strands, correlations, projection of performance and calculation of magnetization at low field.

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